Students' Perceptions on Communicating Mathematically: A Case Study of a Secondary Mathematics Classroom

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Abstract: This paper takes Vygotsky's constructivist point of view to report on the findings of a case study focusing on students' perceptions on communicating mathematically. The study is motivated by the demand for reform and the new curriculum standards for math education that emphasize the importance of conceptual knowledge, reasoning, discourse, and representation (Draper, 2002; NCTM, 2000; etc.). It is also motivated by existing research that calls for a connection to be made between language and mathematics to promote understanding (Hiebert et al., 1996; MacGregor & Price, 1999; Manouchehri & Enderson, 1999; etc.). In this paper, examination of students' perceptions on talking and writing math is situated in the instructional discourse as analyzed on the basis of classroom observation. Specific questions guiding the study include: How do students view the role of talking and writing in relation to different aspects of math performance? How do students' perceptions on talking and writing math serve as dispositions for their performance in talking and writing math? Data were collected through classroom observations, audio taping, and collection of artifacts such as relevant chapters in the textbook, course plans, and student work products including oral interactions and written work. Informal interviews were conducted during the entire period of data collection in the form of "informal conversational interviews" (McMillan & Schumacher, 1989, p.405). Formal interviews were conducted on a voluntary basis and focused mainly on their perceptions about the relationship between communication in mathematics and mathematics learning. At the theoretical level, the study contributes to our understanding of how students perceive the issue of communication about mathematics. Such a perspective may provide researchers and educators with a better focus on the relations between students' understanding and performance in particular communication activities in school math. As a result, the study offers pedagogical implications and directions for classroom actions.

Keywords: Mathematics Education, Communicating Mathematically, Students' Perceptions

Introduction

THIS PAPER TAKES Vygotsky's constructivist point of view to report on the findings of a case study focusing on students' perceptions on communicating mathematically. Specifically, students' views on talking and writing in mathematics will be examined. Through analysis of interview data supported by empirical evidence from classroom observation, the study focuses on the relationship between students' view point and their actual performance in communicating mathematically. The study is motivated by the demand for reform and the new curriculum standards for math education that emphasize the importance of conceptual knowledge, reasoning, discourse, and representation (Draper, 2002; NCTM, 1991, 2000; Smith III, 1996, etc.). The study is also motivated by existing research that calls for a connection to be made between language and mathematics to promote

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understanding (Bickmore-Brand, 1990; Hiebert et al., 1996; MacGregor & Price, 1999; Manouchehri & Enderson, 1999; Monroe, 1996; etc.).

Studies in the area of teaching and learning mathematics reveal that reflection and communication are the key processes in building understanding (Hiebert et al., 1996; MacGregor & Price, 1999; Manouchehri & Enderson, 1999; Monroe, 1996; Warfiel, 2003; etc.). With calls for mathematics reform, language/literacy demands in curriculum standards and an emphasis on communicating about topics in math education (National Council of Teachers of Mathematics, 2000) , mathematics teachers are facing the challenges of both teaching the math content and engaging students in communicating about math. To address the challenge, teachers not only need to have the knowledge and the skills to approach mathematics teaching from both content and communication perspectives, but also need to understand the perceptions of students. Without an understanding of students' perceptions, it would be very difficult for a teacher to effectively motivate the students to socialize into the target discourse.

A growing body of studies in mathematics education has provided detailed accounts of discourse in classrooms (e.g., Simon, 1995; Hiebert et al,1996; Jones &Tanner, 2002; Raiker 2002), resulting in a call for attention to the quality of the classroom discourse and a more interactive approach to teaching mathematics (e.g., Jones &Tanner 2002). However, existing literature offers very little information about students' perceptions on communicating mathematically. In an article published in Communication Education, Rubin (2002) presented a binocular vision for communication education to advocate for research that brings together classroom discourse analyses and studies of outcomes variables relating to social perception. Currently, we still see the same imbalance. Although there are studies on students' perceptions on teaching and learning, most of them are in the field of other content areas such as science education and few are related to communication (e.g., Shymansky, Yore, Henriques, Dunkhase, & Bancroft, 1998; Kent & Towse, 1997; Kempa & Orion, 1996; Palmer, 1999). Within the limited effort made to approach communication in and about mathematics from the students' perspectives is a study by Tanner and Casados (1998) on promoting discussion in math classes in which there is only a brief presentation of a survey collecting feedback from students on reactions and additional ideas for improving discussions. What we see missing is a more extensive research effort linking situated instructional discourse with social perceptions of learners. This paper, by examining the issue of communicating mathematically from the students' perspectives in relation to situated instructional discourse, will hopefully fill some of the gaps in the existing studies.

Theoretical Framework

The general theoretical framework of Vygotsky (1978, 1986) that the study draws upon is the sociocultural and constructivist point of view toward learning. From this perspective, construction of knowledge and development of language are viewed as related in such a way that they require each other to act as catalysts. Furthermore, a constructivist approach to teaching views learning as a result of learners interacting with the environment. In a math classroom, the environment could be composed of participants and all kinds of learning materials. Interaction could take a variety of forms. Classroom activities conducted via the use of language such as talking, reading, and writing constitute some forms of effective interactions. In a study by Raiker (2002) on the use of language by both teachers and learners

in mathematics lessons, it is demonstrated that the use of language, both spoken and written, is crucially related to the learners' construction of mathematical concepts and the subsequent development of mathematical thinking. Also, in the existing literature, it has been argued that articulation in a "culture of mathematising" (Bauersfeld, 1988) provides an opportunity for students to test their understandings; contribute to the generation of corporate meaning; and by listening to others, contrast the interpretation being offered with their own thoughts and modify them appropriately (Clarke, 1994). Existing studies on reflective discourse in mathematics classrooms (Tanner & Jones's 1999, 2000) indicate the importance of a supportive classroom culture where students are expected to articulate and discuss their own methods and conjectures.

While there seems to be a consensus in the research area that communication is important to the development of mathematical understanding, just to "put students in groups and let them communicate as they solve problems" (Richards, 1991) is not very helpful. As argued from the constructivist point of view, "Students will not become active learners by accident, but by design" (Richards, 1991, p. 38). This calling for "designed" communication in mathematics classrooms lays significant emphasis on the teacher's responsibilities in the mathematics classroom which are, as summarized by Cobb, Wood, & Yackel (1993), as fostering the development of conceptual knowledge among her or his students and facilitating the constitution of shared knowledge in the classroom community.

Facilitating the constitution of shared knowledge in a classroom situation places a high demand on the motivation of the students. The essential question to ask is why students, as motivated learners, would want to participate in the constitution of shared knowledge while, learning math in a traditional school setting is a quiet and private process in which students only need to just follow the steps and do it. In another word, while communication as a form for learning is highly valued by researchers and committed teachers, the value of this particular form for learning may not always be obvious to learners. Established learning theories from cognitive and social constructivist point of view clearly indicate that learner factors including purposes and motivation play an extremely important role in a learner's reaction to a learning activity (O'Neil & Drillings, 1994; Volet; 2001; Smith, Dockrell,& Tomlinson, 1997). Thus, it is important to examine students' point of view of a particular learning activity to more effectively implement it.

The Study

This study draws upon data from a larger research project focusing on communicating math. In the original research project, the study (Huang, Normandia, & Greer, 2005; Huang & Normandia, 2005), through the use of functional linguistic approach to discourse analysis, examined students' performance in talking and writing math for the expression of math content related to different levels of cognitive involvement. Findings from discourse analysis reveal that students' performance in communication of math differs depending on the aspects of math content being expressed. Students are more capable of expressing math content at the level of describing actions (e.g., personal choice for method; sequence of the steps; etc.) which is usually less cognitively demanding. They have a much more difficult time when expressing math content at the level of theoretical understanding (e.g., conceptual knowledge necessary for analyzing a problem; reasons behind actions to take; justification of a choice; etc.), which is usually cognitively more demanding. The study also reveals that expression

of math content and sophistication in the use of language in talking and/or writing are interrelated; that the use of specific linguistic features are crucial to expressing desired math content in talking and writing math; that talking and writing math are processes that need to be treated systematically through carefully contrived classroom activities that involve the students in talking and writing math constantly.

In this paper, we shift the focus from the participants' language product to students' perceptions. The analysis of students' perceptions will be situated in the instructional discourse as analyzed in the previous studies. With this effort, we hope to fill the gap in the literature, where there is a lack of studies of students' perceptions on talking and writing math. Specific questions that guided the study include:

1. How do students view the role of talking in relation to different aspects of math performance?
2. How do students view the role of writing in relation to their understanding of the math content?
3. How do students' perceptions on talking and writing math serve as dispositions for their performance in talking and writing math?

The three questions serve to guide the study to link students' perceptions to the findings of discourse analysis of instructional interactions and to seek connections between students' dispositions and the level of performance. Through examining the students' perception on communicating mathematically, we intend to gain a better understanding of talking and writing math from students' perspectives. With such an understanding, we hope to provide instructional implications for classroom teachers to more effectively play the dual roles of fostering the development of conceptual knowledge among her or his students and facilitating the constitution of shared knowledge in the classroom community.

Methods

The study, which employed an ethnographic approach to data collection, is an exploratory one conducted to investigate learner's point of view on certain classroom activities. With a belief that perceptions are always situated in context, we intentionally incorporated various strategies for data collection, minimizing the possibility of gathering information isolated from specific context.

Site and Participants

The study was conducted in a suburban private high school located in central New Jersey. The school administration is proactive in their approach to students' academic achievement and is especially enthusiastic about writing across the curriculum. Teachers in all content areas and disciplines are required to assign writing tasks to students on a regular basis.

All 25 students taking the course from Ms. G. in two different blocks participated in the study. Block 3 had twelve students, while block 4 had thirteen. Among the 25 students, 12 were males and 13 were females. All students were native English speakers and came from middle to higher socio economic backgrounds. In terms of academic achievement, they were average students who were taking the course at the standard pre-calculus level. Parental

support for mathematics education was high and their attitudes were very positive according to these students. In the student interviews, "support" and "pride" are the two words offered most often when describing parental attitudes although direct help from parents in terms of the math content was rare mainly due to the level of the course. The student interviews also indicated a reasonable interest in mathematics, ranging from high to low, with 7 students identifying themselves as highly motivated and only 3 as minimally motivated. Half of the students viewed the importance of the course as college related (11) with only 7 students indicating intellectual challenge as a motivation. Other reasons were related to requirement for graduation, future career, becoming a diverse student, and just being good at it.

Instructional Context

The school is block scheduled. The students take four courses during one semester and meet for 77 minutes in each block. The Mathematics Department of the school had adopted *Advanced Mathematics* by Brown (1994) as the textbook. The course outline indicated that only the first six out of the twenty chapters would be covered in the semester for this group of students.

The students took the course in spring 2003, which for many of them would be the last semester before graduating from high school. By the end of March, the majority were aware of college acceptances. As a result, the social activities outside the curriculum were perceived as more important than academic work in school. Towards the beginning of May, in the last few days of formal instruction for senior students, it was even more apparent that their hearts and minds were no longer in the classroom. Still, they cared about their performance in the course, although the effort demonstrated by several students might not have matched their ability and desire to do a good job.

The typical instructional procedure adopted by Ms. G. was not radically different from the one traditionally adopted by most math teachers: review homework, introduce a new concept or topic, demonstrate how a new concept or topic was used in solving a math problem, have students practice, and assign homework. What was atypical about this class, as observed by us and articulated by the students in the interviews, is that students needed to do a lot of talking. Frequent classroom observations showed that Ms. G.'s math class was "noisy", but the "noise" was mainly the result of conversation consisting of math content. Before this class, at least half of the students had experienced going to the board to demonstrate how they solved a problem, but for all of them, this was the first class in which they had to talk about what they did and why they did it in such a way. While asking the students to "teach the class" or "explain" their way of solving a problem, Ms. G. was particular about having her students use the terminology or jargon accepted in the field of mathematics. To Ms. G., this was a way to socialize students into the discourse of school math. This effort did bring about positive results in terms of helping the students become increasingly proficient in "talking mathematically" about what they did.

During the semester, to address the school's "Writing across the Curriculum" (WAC) requirement, the teacher also involved students in writing mathematically from time to time. While students were constantly asked to describe or explain problem solving in writing while taking unit tests, a formal take-home writing task was also assigned with formal written instruction (see appendix for the assignment). For this formal writing task, the students were given a time period of about two and half weeks to complete the task.

Data Collection

Data were collected between February 18 and May 20 through classroom observations (both classrooms), audio taping, formal and informal interviews, and collection of artifacts such as relevant chapters in the textbook, course plans, and student work products including oral interactions and written work. Audio taping did not start until May 6, 2003 when all consents from the parents were collected.

A total of 51 lessons 77 minutes in length were observed from February 13 to May 20 of 2003. Field notes were taken during on-site classroom observation and were used as a record to identify the relevant content of the lessons to facilitate the selection of usable audiotapes. It also helped to record those events that are relevant but cannot be captured by audio recording. Videotaping would have been a more effective way to capture the process of instruction, but concerns for obtaining parental consents in time prohibited us from using this strategy.

Audio taping was used to obtain classroom oral interactions that occurred as a result of participation in classroom activities. A total of 16 lessons 77 minutes in length were audio taped with the hope that analysis of both the teacher and the students' verbalizations would help reveal participants' use of academic language to talk math. Students' written work was also collected to help the researchers analyze students' writing capability in math. Analysis of data from audio taping and students' written work is used in this study to examine the relations between students' perceptions and their actual performance in communicating mathematically.

Interviews were conducted during the entire period of data collection in the form of "informal conversational interviews", which means "the questions emerge from the immediate context and are asked in a natural course of events" (McMillan & Schumacher, 1989, p.405). This strategy was used to obtain data from both the teacher and the students regarding their backgrounds as well as their perceptions. Some questions used are provided in the Appendix 1.

Formal interviews were conducted with the students on a voluntary basis and focused mainly on their perceptions about the relationship (if any) between communication (including talking and writing) in mathematics and mathematics learning. Twenty-two students participated in the interview either individually or in groups during their lunch break. The following guiding questions were framed to provide a focus for the interview:

1. How do you feel about talking and/or writing in this mathematics class?
2. Do you feel that talking and/or writing have benefited you? How?
3. Do you feel more comfortable expressing your understanding of math content? Examples?

Each interview was audio taped and lasted about 20 to 40 minutes. Copies of assignments and student work included the Writing Across the Curriculum (W.A.C) assignments, class exercises, homework, and tests as part of the data for analysis of students' progress in learning math content and communicating about math topics. The course plans provided an outline for the content coverage and the relevant chapters from the textbook provided a basis for topic analysis.

Data Analysis

To address the questions raised in this paper, data analysis is limited to interview data for participants' perspectives. Published findings regarding students' performance in talking and writing math (Huang, Normandia, & Greer, 2005; Huang & Normandia, 2005) will be utilized to make connections between students' perceptions and the findings of discourse analysis of instructional interactions. We will also explore the connections between students' dispositions and the level of performance.

All audio taped interviews were transcribed. Primarily phenomenological in nature, this study drew on ethnographic approaches as it attempted to describe, identify, and ascribe meaning to the collective and individual experiences of the participants and its emerging culture (Moustakas, 1994; Cresswell, 1998; Thomas, 1993) in the context of integrating communication and math content. Grounded theory informed the design that relied on capturing emergent themes and allowing for their growth and on-going validation (Strauss, 1998). As the researchers were participant observers, the study also benefited from the increased systemic validity characteristic of participatory research (Freire, 1972; Fals-Borda, 1980). Periodic informal and formal discussions among and between researchers and students served as an inductive base from which to identify characteristics of communication activities in students' eyes. Themes of personal reflections were identified through such discussions as well as deepened and expanded through document analysis including researchers' field notes and transcribed interview data.

Findings and Discussion

Discourse analysis of students' talking and writing math from the previous studies (Huang & Normandia, 2003; Huang, Normandia, & Greer, 2005; Huang & Normandia, 2005) revealed that talking and/or writing math does not take place automatically for the students. It requires both time and effort from the teacher and the students. While teacher discourse is rich in expressing a variety of semantic relations associated with various aspects of the content topic function, the semantic relations that the students feel most comfortable expressing are descriptions of actions which are more associated with lower level cognitive skills.

Frequent articulation about math does not indicate effective communication concerning various aspects of a math topic. From a sociocultural perspective (Ochs, 1988), communicating math is basically about learning to communicate and communicating to learn. In this study, we try to find out how students perceive this relation. Findings in the following are situated in the published discourse analysis (Huang & Normandia, 2003; Huang, Normandia, & Greer, 2005; Huang & Normandia, 2005) of instructional interactions.

Student Perspectives on Talking

Among the 22 students who were interviewed, all of them expressed the positive effect of talking on the acquisition and comprehension of math content. In the eyes of the students, talking helped in two directions: self and others. The following themes emerged as findings of the interview on the questions related to talking math. For illustrative quotes, please see Appendix 2.

Question 1: How do you Feel about having Students Talk about Math in the Class?

When addressing this question, several findings emerged. First, having students talk math in the classroom helps create an inviting atmosphere conducive to learning. Second, students feel an increase in understanding while listening to peers explaining. Third, students feel that talking math helped them check for understanding, clarify thinking, and reinforce new knowledge. Finally, some students even raised their math score in this class taught by Ms. G, which involved so much talk from the students.

Question 2: Is it Difficult for you to Talk about Math?

Students' responses to this question lead to the following findings. First, talking math does not come automatically. It was initially difficult for the students even for the kind of talk (discourse associated with articulating sequence of steps) they feel comfortable with now. Second, the experience of talking math has been viewed as a learning process in students' reflections. It was the requirement from the teacher and frequent actual practice that had helped them become more comfortable and fluent while talking. We have observed progress made by students as indicated in the interviews.

Question 3: How do you like the "why" Question?

Though extremely positive towards the activity of talking, the students acknowledged that they were better at sequentially telling about procedures and felt it was difficult for them when being pushed for answers to "why" questions. Our observation and discourse analysis confirmed this perspective.

Students' Perceptions on Writing

The initial analysis of the interview data on writing shows that a number of students felt awkward when asked to write math the first time. Most of the students felt that writing in math was different from doing math in that it was cognitively and linguistically more demanding. They also felt that writing in math was different from writing in English or other social sciences such as history. On the other hand, though they did not enjoy writing math, a majority of the students felt that writing in math has benefited them. Seeing progress they had made, the students were capable of viewing writing in math as a process. One notable finding is that no one was aware of the fact that communicating mathematically was a requirement in the curriculum standard in addition to doing math. When told about this new demand, the students suggested starting math writing in early grades.

Very similar to what Morgan (1994) found in the study focused on writing mathematically, our findings also indicate that students were initially resistant to the task of writing while more willing to talk about what they were doing. Even when feeling confident in solving a particular problem, they claimed to find it difficult to write. However, this study shows the promising sign that students, with support and opportunities for practice, will overcome their initial fear and more readily engage in writing tasks in a mathematics classroom.

Students' Perceptions as Dispositions for their Communication Performance

Analysis of interview data showed that the students' perceptions on communicating mathematically have actually formed positive dispositions for engaging in such communication activities. "Communication involves talking, listening, writing, demonstrating, watching, and so on. It means participating in social interaction, sharing thoughts with others and listening to others share their ideas", (Hiebert et al., 1996, p. 5). It is argued that sharing work "involves more than just demonstrating a procedure; it requires describing, explaining, justifying, and so on" (Hiebert et al., 1996). Our discourse analysis has revealed all these features in communication oriented activities. Furthermore, our classroom observations reveal that regular routine student talk has helped to establish an inviting atmosphere, a crucial factor for the establishment for a "culture of mathematising" which is characterized by subjective, personal reconstruction of knowledge through the negotiation of meaning in social interaction (Bauersfeld, 1988). Analysis of students' perspectives shows that the effects of the interactions serve as the motivation to more active engagement. Interactions in the class "make it easier to learn", "to understand better", "to reinforce", "to catch myself making a mistakes … [and] actually correct yourself", to "help with the vocabulary", to "instill [it in your mind", and to "have an A in the class". As argued by Hiebert et al., (1996), by "communicating we can think together about ideas and problems" and by contributing suggestions, "we often can accomplish more than if we worked alone". Furthermore, while challenging each other's ideas and asking for clarification and further explanation, we are encouraged to "think more deeply about our own ideas in order to describe them more clearly or to explain or justify them" (Hiebert et al, 1996, p. 5 – 6). These effects contribute to the formation of the attitudes in students of seeing learning as "more fun" and wanting "to go to class" and "do more of [my] homework".

Positive dispositions, as reflected in the interview data, do not indicate an easy beginning of the process. In addition, the level of cognitive involvement in student talk varies according to the situation in which student discourse is produced. The students seldom produce discourse expressing reasons behind steps in a leading question and answer dialogue. However, when students are asked to play the role of the teacher to "teach the class" about what they did, which is a routine activity in the class, student discourse becomes more elaborated and carries more features of language associated with higher level cognitive involvement. Playing the role of the teacher, the students seem to have a better chance articulating different levels of understanding.

While all students expressed that they felt much more comfortable talking about procedures, as revealed by the interview data, discourse analysis of classroom interactions showed that initially, the students had to struggle through the process to achieve a certain level of comfort. Our observations show that this comfort was achieved as a result of having frequently engaged in talking about procedures. Through practice, student by student, student talk helped clarify thinking and facilitated talking.

The examination of student perceptions also indicates a high level of discomfort about addressing the "why" questions (more related to conceptual understanding) in the class even they show an appreciation of "why" questions as helpful in facilitating understanding. Discourse analysis of classroom interactions seem to be able to offer an explanation why this is the case. Among a total of 1,232 minutes transcription from 16 lessons, the only places

where we could find elaborated student discourse addressing "why" questions" are when the students were invited to "teach" the whole class about how they solved a problem. This kind of interaction takes about 10 to 15 minutes of each lesson of 77 minutes which is less than 20% of class time. Though constant active interaction among participants is a characteristic of the class, most of the time interactions are in the form of leading question and answer dialogue (Gregg, 1995) where student discourse is limited to articulation with less demanding cognitive involvement. From time to time, the teacher did try to have students articulate at higher thinking levels. It is obvious from our discourse examples that the teacher is aware of the importance of pushing for student discourse that involves higher-level thinking. Even though she consciously makes an effort to have this realized the result is not consistent. As revealed in our studies focusing on discourse analysis, in one occasion, a student did not achieve using his conceptual knowledge about absolute value to justify a solution. The teacher tried to push the student to use the basic concept to justify a solution by guiding the student to refer to the definition of absolute value. In most cases, the student talk remains at the level of talking about procedures. Sometimes, the teacher pushes for it and the student discourse, in response to the teacher's elicitation, shows linguistic features for expressing conceptual knowledge, but eventually it is the teacher who provides the more accurate and comprehensive discourse without inviting the students to practice. Some other times, the teacher tries to have the students articulate reasoning to justify the solution but did not insist when the answer given by the students was correct. While the students would be encouraged to produce and would be praised for the desired discourse, it is not explicit and systematic for the students that all solutions should be backed up by the basic concepts or reasoning. Though the teacher tries to push for the use of concepts or reasoning, no such requirement was made explicit for the students. Thus, we suspect, students end up performing inconsistently. It may also explain the students' discomfort with addressing the "why" questions.

Students' perceptions on writing math also provide us with insights on the role and implementation of writing activities from the students' perspectives. From the students' perspectives, writing math is a form of communication that can help students understand better. Obviously, the students are also capable of viewing writing as a process in which progress is gradually made. One salient point made by the students is that writing math is different from writing in History or English and proficient writers may not feel as comfortable when it comes to writing math. Reasons for this discomfort about writing math are provided by the students in several categories: 1) use of different vocabulary that they "are not familiar with"; 2) use of different text patterns that are more "based on steps that you follow"; 3) expository writing rather than creative writing where "your own expression" gives way to providing "more information in your explanation"; and 4) use of different semiotics where "you need to think about both words and numbers and try to explain your thoughts".

Our discourse analysis of written texts by students reveals some parallels between students' perceptions and characteristics of student writing (Huang & Normandia, 2005). In our comparison of the teacher's text and two student texts (a stronger and weaker one), we find differences in both meaning construction and linguistic realization. The topic of the assignment requires the students to demonstrate their conceptual understanding of the *discriminant* and procedural knowledge as the basis for a determination of the nature of the roots of a quadratic function. Our discourse analysis shows that both the teacher text and the stronger student text successfully demonstrated the construction of conceptual knowledge and utilized reas-

oning to justify the steps they show in the informative explanation. On the other hand, the weaker student text did not achieve the purpose of demonstrating an understanding of the basic concepts involved and provided no reasoning to form an informative explanation. There is also a difference in the utilization of linguistic resources for knowledge construction and demonstration. In the more successful texts, accurate use of vocabulary and a skillful use of combination of numbers and words play a vital role in effectively constructing the knowledge involved. The weaker text is problematic in both meaning construction and utilization of linguistic resources. First, for construction of conceptual knowledge, use of linguistic devices such as lexis (vocabulary) plays a crucial part in demonstrating conceptual understanding. In the weaker text, erroneous or vague use of crucial terms reveals a lack of conceptual understanding of the terms. Also, the choice of lexis indicating part-whole relations exhibits confusion in conceptual understanding which is vital for writing informational texts.

The discriminant is the number and the radical in the quadratic formula .
The discriminant is the number and the radical in the quadratic formula .

In addition, very crucially for meaning construction, there is a lack of use of linguistic features associated with reasoning which is a requirement for constructing explanation.

In connecting the findings from discourse analysis and student perceptions, we are truly amazed by the students' capability to identify the features in mathematics writing. Student perceptions enable us to see the issue from the student perspectives while discourse analysis provides us with the context in which student perceptions are situated. Both sets of data point to similar implications. A stronger written text in math expresses more semantic relations, which means it is more informative in the explanation. Also, stronger texts establish required semantic relations with clarity by using more varieties of linguistic features, which includes the use of vocabulary and various semiotic systems for expression.

Summary and Conclusions

In traditional mathematics education, instruction is based on the conception of "teaching children to be doers rather than thinkers" and thereby "considerably underestimates what children can learn" (Fuson, 1992, p. 57). A lot of attention is paid to the students' ability to follow the procedures to do math. Nevertheless, being able to follow the procedure at the moment of instruction, even in the case of arriving at a correct answer, does not necessarily indicate an understanding of the concept. A lack of understanding of the reasons behind the procedure may very likely result in an erase of the procedure from the memory soon after the instruction (see Bass, 2003; Davis, Hill, & Smith, 2000; Van de Walle, 2007). We suspect this is one of the reasons that the math content does not really stay with the students after the instruction. Student interviews indicate that verbalization of theoretical understanding for actions help them achieve understanding at this level.

Our discourse analysis shows increased capability of students to communicate mathematically in both talking and writing over a period of time. However, the interview data indicates a conflict between student's knowledge about talking or writing and their desire to talk and write mathematically. The responses from the students provide implications for classroom practice. The students all agree that talking and writing are useful and effective at different levels in helping them think and understand better, but most of them do not enjoy this. Many

of them were very nervous about having to talk or write about math, however, they feel much more comfortable after being required to do this for a period of time though it does not mean they enjoy writing. This indicates that our students are mature enough to appreciate the instructional strategies intentionally planned by the teacher. Teachers need to insist on effective strategies even if facing resistance initially.

Being persistent does not mean that we need to force the students without justification. When being asked if they knew that communication about mathematics is one goal in today's mathematics education, all students expressed that they did not know. Quite a number of them expressed surprise because they have never viewed the capability of communicating mathematically as an indication of being good at math. They still view being able to do math quickly without struggle as being evidence of mathematical competence. However, when being told that this is a requirement, they expressed that this should have started at the elementary level, just as we do in language arts. We were impressed and believe that these insights should provide all math educators with implications that should be taken seriously. We need to communicate the dual goals of mathematical education to the students explicitly: doing math to solve problems and communicating mathematically.

The study is an attempt to examine communication about mathematics in a school setting from students' perspective. Published findings on discourse analysis are used to contextualize the interview data. At the theoretical level, the study contributes to our understanding of how students perceive the issue of communication about mathematics. Such a perspective may provide researchers and educators with a better focus on the relations between students' understanding and performance in particular communication activities in school math. As a result, the study offers pedagogical implications and directions for classroom actions. Analysis of student interviews situated in discourse analysis of classroom interactions strongly indicates that talking and writing is important to mathematics learning. It is not a waste of time. A specific requirement for practice is important. Students will listen if they know the goals of learning and are willing to put in effort and participate if the connection between the goals and activities are made clear to them.

More questions are raised from this study. 1) How could we better communicate to the students the importance of communication in school math? 2) Since communicating in math is different from communicating in other content areas, how could we design our instruction to better help the students systematically improve their communication skills in school math? 3) Would this improvement in communication skills foster the conceptual understanding (and therefore long term memory) that is the outcome sought (or should be) in mathematics education? 4) Even with an understanding of the importance of communicating in math, students are still reluctant to participate in communication activities. Then, how could we effectively motivate the students to be more involved in this? Answers to these questions may provide useful instructional implications at all school levels.

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Appendix 1. Questions used in informal interview:

1. Ethnic background:
2. Do you speak English as your first language?
3. If English is not your first language, what is your first language?
4. Do you speak another language fluently?
5. What motivates you to keep taking math?
6. Have you always been strong in math? Please explain.
7. Have you always been weak in math? Please explain.
8. Do you get help in math from family when needed? What kind of help?
9. How do your parents view your studying math?
10. How do you feel about the math you are taking? Easy? Interesting? Useful? A welcome intellectual challenge? Fun? Why do you feel so?
11. How well prepared do you feel you were prior to taking this course, i.e., "Analysis of Functions"?
12. Do you consider yourself as a good writer? Please explain.

Appendix 2

Question 1: How do you Feel about having Students Talk about Math in the Class?

Findings	*Illustrative Quotes*
1.1. Having students talk math in the classroom helps create an inviting atmosphere conducive to learning.	*Derek: "It is better because it's not that boring. Math in general, for young people is so boring, but when you get to talk more, and interact more, it makes it easier to learn."* *Derek: "You want to go to class because you don't just sit there like a zombie, you actually interact and you learn more. It keeps you awake, and you are like "Oh my God, I want to go to class." You think before you speak, which makes you think more."* *Derek: "It is more fun to learn."*

1.2. Quite a number of students expressed an increase in understanding while listening to peers explaining.	*Michelle: "Some students are visual learners and some are audible learners. It helps to be able to hear and say it."* *Kristen: "sometimes having someone else explain it, makes it easier to understand. Peers help a lot."* *Jenna: "A lot of people yell out questions, which helps me because it might have been something that I was wondering as well."* *Jenna: "She makes us explain so that if others don't understand maybe they could learn through us and it also checks to see what we know as well. By the end of a lesson you have heard it 3 or 4 times, in different ways."* *Pete: "The talking in the class makes me feel more comfortable to ask questions if I do not understand something."*
1.3. They expressed how talking helped them check for understanding, clarify thinking, and reinforce new knowledge.	*Kristen: "...you have to teach yourself it. It is not just like reading a story, you actually have to put thought into it."* *Michelle: "I don't like it at all but I feel I understand it better because I have to speak it, almost proving to myself that I know the problem."* *Louis: "It is a reassurance that I am right and understand it better because I have to speak it, almost proving to myself that I know the problem."* *Eddy: "Yes, in order to understand it you have to talk with the other students. It makes us understand it more."* *Megan: "It reinforces what you are learning and I feel that I am learning it better because I have to say it so when I do the next problem I explain the steps in my head."* *Tom: "Sometimes I catch myself making a mistake but when you talk it out and try to explain it you realize right away. You actually correct yourself."* *Ashley: "It instills it in your mind."*

	Kate: "It reinforces the problem, and helps with the vocabulary." *Derek: "I think that anytime, just having to go to the board, and then having to explain it, it reinforces it in your head."*
1.4. Some students even mentioned how this class taught by Ms. G, which involved so much talk from the students has helped raise math score	*Jason: "100%. This is the best math class I have had in the four years I have been here, and it's the best grades I have had as well. Because it was more fun, and you talk and interact. I guess it works because my grades have gone up; I have an A in the class."* *Jason: "I do more of my homework because I understand it more, and I get it done a lot faster too."*

Question 2: Is it Difficult for you to Talk about Math?

Findings	*Illustrative Quotes*
2.1. Talking math does not come automatically. It was initially difficult for the students even for the kind of talk (discourse associated with articulating sequence of steps) they feel comfortable with now.	*Derek: [When asked if he resisted talking] "A little bit."* *Kristen: "Yes, but it is still hard to use the terminology because you forget and use words that you have used since you were little, basic terms, but now you have to use bigger terms."* *Megan: "I wasn't used to it, but it got easier."* *Pete: "I was nervous."*
2.2. The students also reflected on the experience as a learning process. It was the requirement from the teacher and frequent actual practice that had helped them become more comfortable and fluent while talking. We have observed progress made by students as indicated in the interviews.	*Derek: "As you do it, it gets easier and you see the point."* *Kristen: "It sticks in you mind when you have to use them over and over."* *Megan: "I wasn't used to it, but it got easier."* *Tom: "The more you talk about it, the more you remember come test time"* *Ashley: "Yes, it is like a routine now, or like a habit. The more you do it, the more comfortable you become."* *Pete: "The practice helps."*

	Michelle: "I feel better now but it was difficult at first. I have been almost teaching my mother how to do these problems when I ask for help and I feel that I learn just by doing that. Talking helps me a lot"

Question 3: How do you Like the "why" Question?

Findings	*Illustrative Quotes*
Though extremely positive towards the activity of talking, the students acknowledged that they were better at sequentially telling about procedures and felt it was difficult for them when being pushed for answers to "why" questions. Our observation and discourse analysis confirmed this perspective.	*Megan: "I don't like why questions but they help you understand better; it makes you understand why you just did what you did"* *John: "It is hard for us to explain why, that is why we have the teacher, for her to explain."* When asked if they would like to explore the why questions: *Megan: "Yes."* *Anthony: "Why not?"* *Aaron: "It (the why questions) is good to know the answers (to the why questions).*"

Students' Perceptions on Writing

Several Themes Emerged from the Initial Analysis of the Interview Data on Writing as Shown in the following

Themes	**Illustrative Quotes**
1. Quite a number of students felt awkward when asked to write math the first time.	• *L: I was surprised that we had to do this.* • *T: It was weird at first having to write about the problems.* • *C: You think of it as more an English thing than a Math thing.*

2. Most of the students felt that writing in math was different from doing math in that it was cognitively and linguistically more demanding.	• *J: It is a lot different to have to explain in words, and you have to sit there and think for a while. You have to be very precise about what you are saying. You actually have to stop and think.* • *D: you actually have to write it out and instead of using numbers; you have to use words to explain. Forcing you makes you understand it because the words are right in front of you.* • *K: It is harder because usually you just do it and don't have to explain it. You know how to do it mathematically but it is hard to write it down on paper properly.* • *Mike: It is just numbers so it is hard to look at it from a literary point of view. I felt overwhelmed.* • *Keith: To be able to figure out how to write the Math problems out in complete sentences, it is like a brick wall. It is not like you can look up how to write in Math like you would if you did not understand how to do a problem. You have to fully understand the problem before you can go and write about it. The sequence of steps is easy but having to explain the concepts is difficult. You need to know the vocabulary and a lot of new concepts. Letters instead of numbers, it is more abstract and challenging.* • *Pete: When having to explain why, the wording in your explanations needs to be understood prior. There is a lot more thinking involved, knowing the steps is just not enough.*
3. Though they did not enjoy writing math, a majority of the students felt that writing in math has benefited them.	• *John: It helps explain it more.* • *Kate: I learned vocabulary that I had never used before that I needed to know in order to describe what I was writing about. It is hard but you learn. It is clearer now because writing it out helps clarify. It is words instead of just numbers.* • *Keith: Writing pushes you to understand the concepts. I think everyone did a lot better because the concepts were really drilled into your head.*

	• *Louise: It is like reassurance for me and knowing that what I am doing is right. You actually have to put it into words, which helps me remember it later on.* • *Tom: Instead of thinking in terms of just numbers which your mind can go kind of fast, writing about it forces you to slow down and think about it and it is easier to pick up mistakes.* • *Kristen: this year I didn't understand the problem so now after having written about it and figured it out I will never forget it. Now I am clear on the problem. It kind of forced me to understand.*
4. Seeing progress they had made, the students were capable of viewing writing in math as a process.	• *Pete: The first year was difficult but eventually you get used to it and it is not as bad.* • *Michelle: I am better now but I was always a writer just not for Math.* • *Kate: We never had to do this growing up so it is difficult but we are getting used to it now.* • *Aaron: I feel myself getting better. I try to transfer the skills from English to Math writing.* • *John: I am used to writing in English class but not used to writing in Math class. It's just new.*
5. No one was aware of the fact that communicating mathematically was a requirement in the curriculum standard in addition to doing math. When told about this new demand, the students suggested starting math writing in early grades.	• *Courtney: It would have been different if I had started at a younger age.* • *Mike: The fact that you only do it once a year doesn't help (once a course). If you did it more than once, you might understand it better. In English we write a lot more and it is easier.* • *Kristen: I agree (with Mike), if we had to do it more it would be easier where as Math is not sometime that we generally have to write for. So you struggle more.*
6. Most of the students felt that writing in math was different from writing in English or other social sciences such as history.	• *Keith: Skills can be transferred. I am not bad but I am not good. With English writing and History it is story related or you are given a topic. If it is a topic I am familiar with or can find information on it, and not have to read in between the lines. Grammar is different. The way you have to write in Math is based on steps that you follow.*

	• *Tom: I enjoy writing but I prefer to write about topics that I enjoy and Math is not one of them. It is difficult because you need to think about both words and numbers and try to explain your thoughts.* • *Kate: I am good at writing about things I enjoy about because I am a free writer but Math is different because it is more complex because of vocabulary I am not familiar with.* • *Ashley: I am a good writer in English but that is different because you are writing about opinions and in Math you are writing about facts. The vocabulary in Math I don't really use.*

About the Authors

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Growing up and educated in China, further educated and "polished" in Canada, and currently teaching and researching in an American University, I have cultivated in myself the sensitivity to language issues across academic settings. With a Ph.D in Curriculum and Instruction from University of British Columbia, I am currently working as an Associate Professor of language education, the Chair of the Curriculum and Instruction Department, and the University Coordinator for the On-campus LMS Support Services. Teaching courses in the areas of ESL education, foreign language education, diversity in the classroom, and content literacy across the curriculum, I have been doing research in all these areas focusing on classroom discourse, integration of language and content, and mainstreamed ESL students at all levels. Recent publications include articles in Communication Education; Language and Education; Linguistics and Education; Language, Culture, and Curriculum; International Journal of Learning; International Journal of Applied Linguistics; Language Teaching Research. Most recently, I am working on the topic of "Talking Math" and "Writing Math".

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